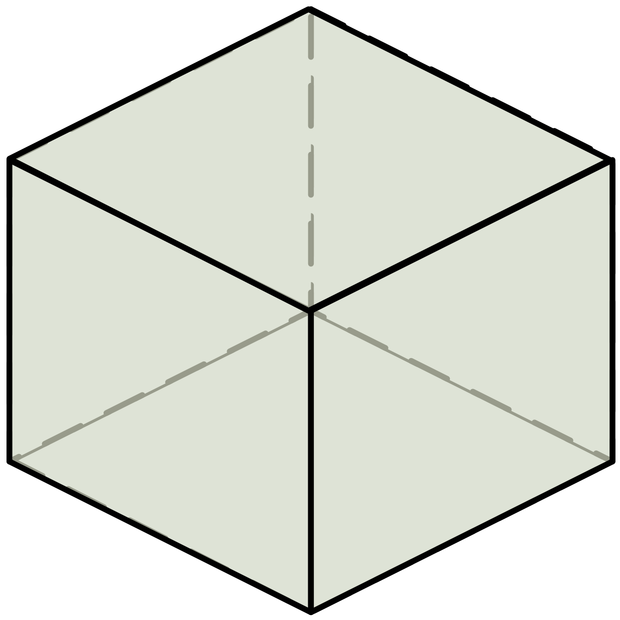
Algebra with Cuboids

Beneath is a picture of a **cuboid**. It has four vertical edges, four edges going from bottom right to top left and four edges going from bottom left to top right. Altogether, the total number of **edges** in a cuboid is 12.



Where the edges meet, there is a corner. This corner is called a **vertex**.

There are eight **vertices** (plural of vertex) in this cuboid.

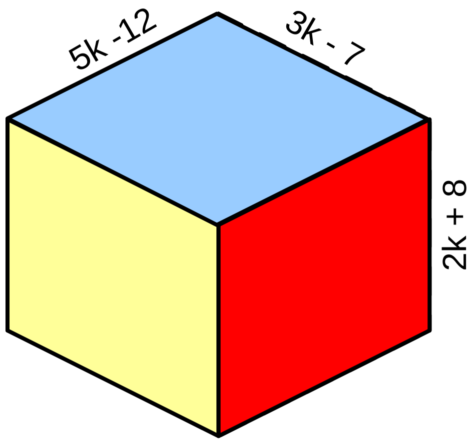
Four edges and four vertices form a series of rectangles around the edge of the cuboid. These are called the sides of the cuboid.

Opposite **sides** of the cuboid are the same. The top and bottom sides are the same as each other. The left and right sides are the same as each other. The front and back sides of the cuboid are the same as each other. There are six sides to a cuboid.

This help sheet will tell you how to calculate three values referring to a cuboid. These are:

- Length of the edges;
- Surface area;
- Volume.

Worked Example

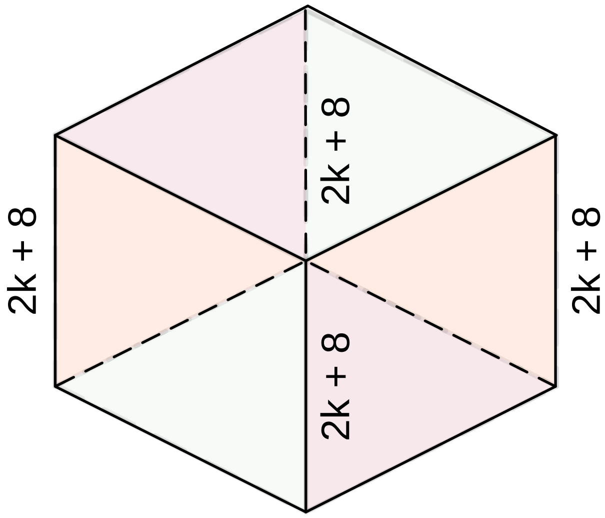


For the cuboid shown, calculate the:

- **Length of the edges;**
- **Surface area;**
- **Volume.**

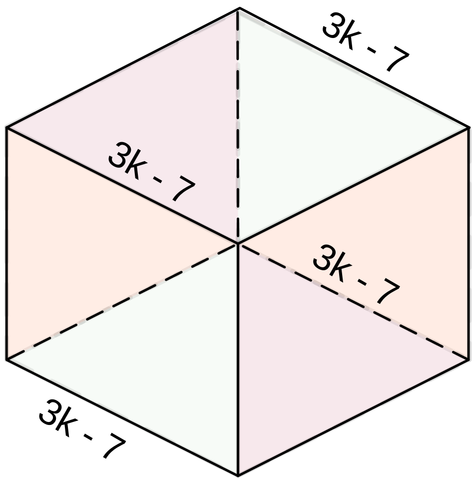
To Calculate the Length of the Edges:

Calculating the lengths of the edges would be important if you were an electrician and you needed to workout what length of wire you might to install a light fitting and switches.

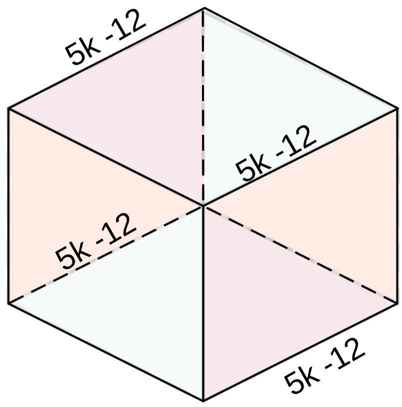


The diagram above shows the individual lengths of the vertical parts of the cuboid. Each of these lengths is 2k+8 cm long. Altogether, there are 4 lots of (2k+8) and so we can write that as **4(2k+8)**.

As an alternative, we could also put **2k+8+2k+8+2k+8+2k+8**. Either of these is just as valid but we only put one of them.



In the same way, there are four sides, each of which is (3k-7) cm long. Again, we can write these in one of two ways but it is important that you only include one of them in your calculation. We could either write **4(3k-7)** or we could write **3k-7+3k-7+3k-7+3k-7**. The first method is using brackets to multiply the length by four. The second method is just adding the first side to the second side to the third side to the fourth side. Both are valid, but again, only choose one.



The final sides are now marked upon the diagram. Again, there are four of them. Each one, (5k-12) cm long. So the total of these final sides is **4(5k-12)** or **5k-12+5k-12+5k-12+5k-12**.

Now it is time to put all this information together. I will show you how to perform the calculations using both the addition and the multiplication methods. You need to choose one of these and do that.

Using the addition method:

Total length of edges, T_{edge}

T_{edge} = 2k+8+2k+8+2k+8+2k+8 + 3k-7+3k-7+3k-7+3k-7+5k-12+5k-12+5k-12+5k-12

= 8k + 32 + 12k - 28 + 20k - 48 (Just add up the ks and numbers)

= 40k - 44 cm

The whole point with this method is to add the ks and the numbers separately until you have a final number for k and a final number. Be very careful with whether the number is a plus(+) or a minus(-) sign.

Using the multiplication method:

T_{edge} = 4(2k+8)+4(3k-7)+4(5k-12)

= (8k+32)+(12k-28)+(20k-48)

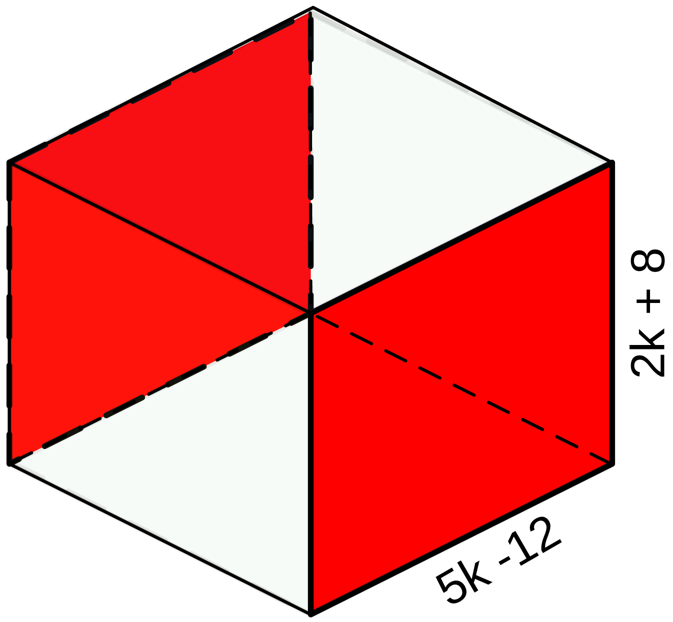
= 40k - 44 cm

Both methods lead to the same answer. This is particularly pleasing because if they didn't, one or both of the answers would be wrong.

Explaining How To Calculate the Surface Area:

Calculating the surface area is important because it is needed to see how much material you would need to make the cuboid (eg in the form of a box or a room etc).

There are six sides altogether in a cuboid. You need to be able to work out the area of three of these sides and double it to find the overall area of the cuboid.



The above diagram shows two of the sides and gives the dimensions in terms of k. For a minute, imagine that the height (or breadth) of the rectangle was 8 cm and the length of the rectangle was 12 cm. You would calculate the area of the rectangle in the following way:

$A_{Rectangle}$ = length × breadth

= 12 × 8

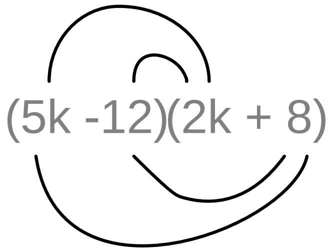
= 96 cm^2

With the use of algebra, the rule for calculating the area of this side is exactly the same, but the logistics of following it are a little more complicated.

Firstly, you need to remember the smiley face that we drew for multiplying out brackets.

$$(5k - 12)(2k + 8)$$

... means that you have to multiply 5k × 2k, 5k × (+8), 2k × (-12) and (-12) × (+8).



Below is the worked example of how to calculate the area of one red side:

A_{Red}	=	length × breadth	**(0)**
	=	$(5k - 12)(2k + 8)$	**(1)**
	=	$(5k \times 2k) + (2k \times (-12)) + (5k \times (+8)) + ((-12) \times (+8))$	**(2)**
	=	$10k^2 \quad + \quad (-24k) \quad + \quad 40k \quad + \quad (-96)$	**(3)**
	=	$10k^2 \quad + \quad (40-24)k \quad + \quad (-96)$	**(4)**
	=	$10k^2 \quad + \quad 16k \quad + \quad (-96)$	**(5)**
	=	$10k^2 + 16k - 96$	**(6)**

Explanation of what I have done

On line (0), I have set out what I am going to do, ie calculate the area by multiplying length by breadth.

On line (1), I have put in the terms for length and breadth so I have swapped the word length with (5k-12) and swapped the word breadth for (2k+8).

On line (2), I have written down all the calculations that I am actually going to perform. I have put add signs between each one, even though, I know that some of them are going to end up as minus terms. I have put the sign and term, eg (-12), in parenthesis as that keeps the number together and helps me to think.

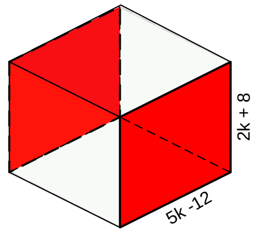
On line (3), I have done each multiplication in each set of parentheses and then written down what I got. I put plus signs between each one, even though I know some are minus to help me to think.

On line (4), I have grouped together the k terms. Note that k^2 is not the same as k.

On line (5), I have calculated the k term.

On line (6), I have written the equation as it should be written noting that I have disposed of the last + sign and put a minus (as a minus times a plus is a minus).

The amazing bit (This is a good check but is for interest only):



If we said that k was worth 4, then our rectangle would have a length of 5(4)-12 = 8cm and a breadth of 2(4)+8=16cm.

A_{Red} = length × breadth
= 8 × 16
= 128 cm^2

Using the formula that we have calculated above, we have:

A_{Red} = $10k^2 + 16k - 96$

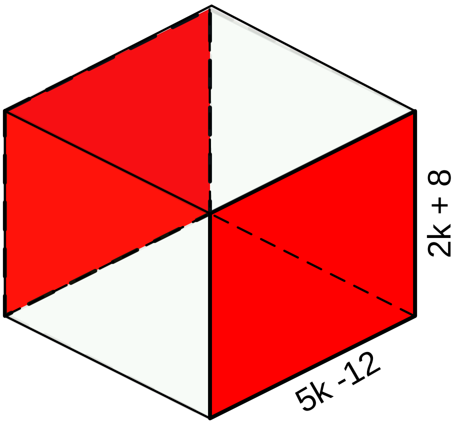
= 10(4×4) + 16(4) - 96

= 160 + 64 - 96

= 128 cm^2

Now we have gone through the method for multiplying one bracket by another, we need to use this method to calculate the surface area of the cuboid.

To Calculate the Surface Area of a Cuboid:



A_{Red} = 2(length × breadth) (2 is there as there are 2 sides)

= 2((5k-12)(2k+8))

= 2((5k × 2k) + (2k × (-12)) + (5k × (+8)) + ((-12) × (+8)))

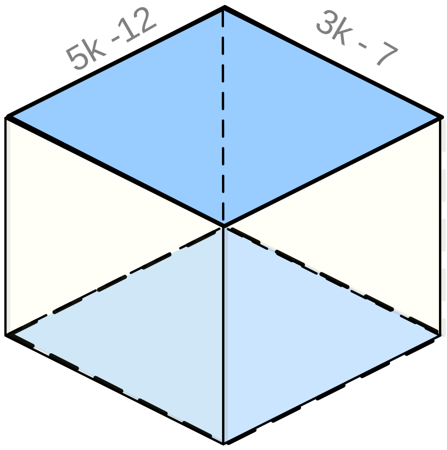
= $2(10k^2$ + (-24k) + 40k + (-96))

= $2(10k^2$ + (40-24)k + (-96))

= $2(10k^2$ + 16k + (-96))

= $2(10k^2$ + 16k -96)

= $20k^2$ + 32k -192



A_{Blue} = 2(length × breadth)

= 2((5k-12)(3k-7))

= 2((5k×3k)+(5k×(-7))+(3k×(-12))+((-12)×(-7)))

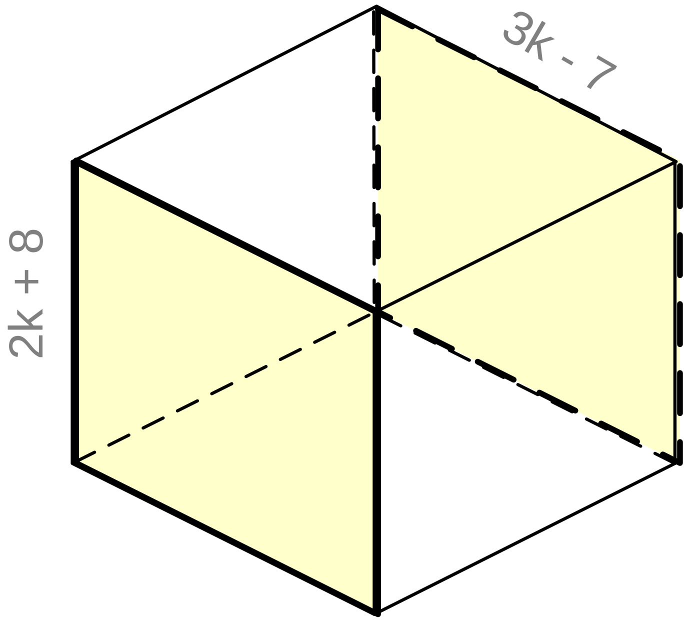
= 2($15k^2$ + (-35k) + (-36k) + 84)

= 2($15k^2$ + (-35- 36)k + 84)

= 2($15k^2$ + (-71)k + 84)

= $2(15k^2-71k+84)$

= $30k^2 - 142k + 168$



A_{Yellow} = 2(length × breadth)

= 2((2k+8)(3k-7))

= 2((2k × 3k)+(2k × (-7)) + (3k × 8) + (8 × (-7)))

= $2(6k^2 + (-14k) + (24k) + (-56))$

= $2(6k^2 + (24-14)k + (-56))$

= $2(6k^2 + 10k + (-56))$

= $2(6k^2 + 10k -56)$

= $12k^2 + 20k - 102$

To calculate the total surface area of the cuboid, we need to add these things together and group like terms (which means putting all the k^2, all the k and all the numbers together).

A_{Total} = A_{Red} + A_{Blue} + A_{Yellow}

= $(20k^2 + 32k -192) + (30k^2 - 142k + 168) + (12k^2 + 20k - 102)$

= $(20+30+12)k^2 + (32-142+20)k + (-192 + 168 - 102)$

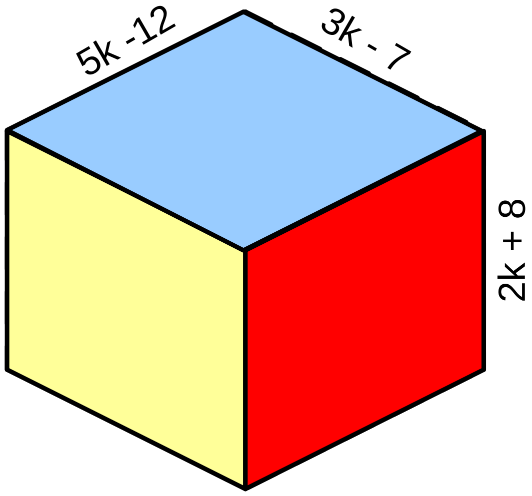
= $62k^2 - 90k -126$

Which gives the total surface area of the cuboid in terms of k.

To Calculate the Volume of the Cuboid:

To calculate the volume of a cuboid, the formula is as follows:

$$V_{Cuboid} = \text{length} \times \text{breadth} \times \text{height}$$



For this particular cuboid, we have length = (5k-12), breadth = (3k-7) and a height of (2k+8).

Just as if you were multiplying the sides 3cm, 4cm and 5cm to find the overall volume, you would choose 2 and do them first, we do the same here.

V_{cuboid} = length × breadth × height

= (5k-12)(3k-7)(2k+8)

= $(15k^2 - 71k + 84)(2k + 8)$

= $(15k^2 \times 2k) + ((-71k) \times 2k) + (84 \times 2k) + (15k^2 \times 8) + ((-71k) \times 8) + (84 \times 8)$

= $30k^3 \quad -142k^2 \quad + \quad 168k \quad + \quad 120k^2 \quad - 568k \quad + \quad 672$

= $30k^3 - 22k^2 - 400k + 672$

Which is the answer.

Important things that you will have learnt or revised by doing this exercise:

1. **Set your work out neatly.**
2. **How to multiply brackets when algebra is included.**
3. **Grouping like terms together.**